MATH 306 Workshop

Important Theorems: (you should also review all the definitions)
Eigenvector: 5.10
Upper-triangular Matrix: 5.27, 5.30, 5.32
Fundamental Theorem of Algebra

1. Write down the definitions:
    a. Invariant subspace

    b. Eigenvalue

    c. Eigenvector

    d. Upper-triangular matrix

2. Conditions equivalent to diagonalizability. (Thm 5.41) **Important**

    a. T is diagonalizable.

    b.

    c.

    d.

    e.

3. Let T $\in \mathcal{L}(V)$, prove that Null T is invariant under T.

λ

4. Prove or disprove: Suppose $T \in \mathcal{L}(V)$ and V is finite dimensional, T has an eigenvalue implies $T - \lambda I$ is not injective.

5. Let T (w, z) = (z, z) in $\mathbf{F}^2$. Find the eigenvalues and eigenvectors corresponding with each eigenvalue.

6. Prove or disprove: Suppose V is finite dimensional. If $T \in \mathcal{L}(V)$, then T has at most n eigenvalues where n = dim V.

7. Suppose $T \in \mathcal{L}(V)$ and $T^2 = I$ and -1 is not an eigenvalue of T. Prove that T = I.

8. Suppose $T \in \mathcal{L}(V)$ and $(T - 2I)(T - 3I)(T - 4I) = 0$. Suppose λ is an eigenvalue of T. Prove that $\lambda = 2$ or $\lambda = 3$ or $\lambda = 4$.